MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

BMS1024 – MANAGERIAL STATISTICS

(All sections / Groups)

11 MARCH 2020

9.00 a.m – 11.00 a.m

(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FOURTEEN (14)** printed pages with:
   Section A: Ten (10) multiple choice questions (20%)
   Section B: Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restrictions.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.

1. Given a set of data contains five values:

   3.4 4.7 1.9 7.6 6.5

   Determine the median:
   A. 4.7
   B. 1.9
   C. 3.4
   D. 6.5

2. You were told that the first, second and third quartiles of female students' weight at a major university are 45 kg, 60.2 kg and 74.8 kg. What is the percentage of the female students' weight more than 74.8 kg?
   A. 75 percent
   B. 25 percent
   C. 50 percent
   D. 74.8 percent

3. A company sells annuities must base the annual payout on the probability distribution of the length of life of the participants in the plan. Suppose the probability distribution of the lifetimes of the participants is approximately a normal distribution with a mean of 68 years and a standard deviation of 3.5 years. What proportion of the plan recipients would receive payments beyond age of 75 years old?
   A. 2
   B. 0.9772
   C. 0.0228
   D. 1

4. Based on a sample of data, the calculated mean is 105.5 and the median is 105.5. State the shape of skewness for this data sample.
   A. Positive Skewness
   B. Left Skewness
   C. Right Skewness
   D. Symmetrical

Continued...

5. A set of data below represents the number of cargo manifests approved by customs inspectors of the Port of New York in sample of 10 months:

17	18	18	22	22	22	22	24	24	24

State the mode of this given data:
A. 22
B. 24
C. 18
D. 17

6. The rate of return (%) of an Internet Service Provider over a 6-year period are listed as below. Calculate the mean rate of return:

10.25 12.64 42.53 21.52 -2.35 8.37

A. 16.28%
B. 15.49%
C. 92.96%
D. 21.52%

7. The following examples are described as a discrete data except __________.
A. number of children per family
B. flipping a coin
C. a person's height
D. outcome of rolling a fair dice

8. The following data represents the number of vitamin supplements sold by a health food store in sample of 10 weeks:

19	20	20	22	23	25	30	33	35	38

Determine the inter-quartile range of this data.
A. 35
B. 33
C. 20
D. 13

9. The probability for the cumulative standardized normal distribution at $1.5X$ is 0.9332. The value of X is _________.
A. 1.00
B. 1.50
C. 0.50
D. 0.10

Continued...

10. The average score for first online quiz of a statistics subject is 7.5 and the standard deviation is 2.8865. The relative dispersion for this first online quiz is

A. 7.5%
B. 38.49%
C. 2.8865%
D. 25.98%

Continued...

SECTION B: STRUCTURED QUESTIONS (80 MARKS)

There are THREE questions in this section. Candidates MUST answer ALL THREE questions.

Question 1 (25 Marks)

(a) According to a poll conducted in year 2001, 52% of American adults think that protecting the environment should be given priority instead of developing United of States energy supplies. Thirty-six percent think that developing energy supplies is more important, and 6% believe the two opinions are equally important. The rest had no opinion. Suppose that a sample of 20 American adults was quizzed on the subject.

(i) What is the probability between eleven until thirteen American adults think that protecting the environment should be given priority?

(4 marks)

(ii) Determine the probability at least three adults think that developing energy supplies is more important.

(4 marks)

(iii) Calculate the expected number of American adults believe that two opinions are equally important.

(3 marks)

(iv) Find the probability of less than two adults had no opinion.

(3 marks)

(b) A shopping mall estimates the probability distribution of the number of customers, x actually enters the mall. The probability distribution is provided as below:

x	0	1	2	3	4	5	6
Probability	0.04	0.19	0.22	0.28	0.12	0.09	0.06

(i) Determine the probability at least four customers actually enter the mall.

(3 marks)

(ii) Calculate the expected number of customers actually enter the mall.

(3 marks)

Continued...

(iii) Determine the variance and standard deviation for the above random variable.

(5 marks)

Question 2 (25 Marks)

(a) Dr. Annie Fox is an industrial psychologist. She is studying stress level among executives of internet companies. She has developed a questionnaire that she believes measures stress level. A mean score more than 80 indicates stress at a dangerous level. A random sample of 15 executives revealed the following stress level scores:

64	78	83	80	68	99	97	97
75	93	94	100	75	84	70	-

(i) Construct a 90 percent confidence level for the population mean of stress level scores among executives of internet companies.

(7 marks)

(ii) Is there enough evidence at the 5% significance level to conclude that internet executives have a mean stress level at the dangerous level, according to Dr. Fox's test?

(8 marks)

(b) According to a year 2011 survey of college freshmen in Pahang, 39.5% of freshmen said that they had spent 6 or more hours a week, for studying. This percentage was 37.3% in the year of 2010 survey of freshmen by the same state, Pahang. The sample sizes for this survey was 2000 freshmen in year 2010 and 2200 freshmen in 2011.

Test whether the proportions of college freshmen in 2010 spent 6 or more hours a week for studying, is smaller than the proportions of freshmen in 2011. Test at 5% significance level.

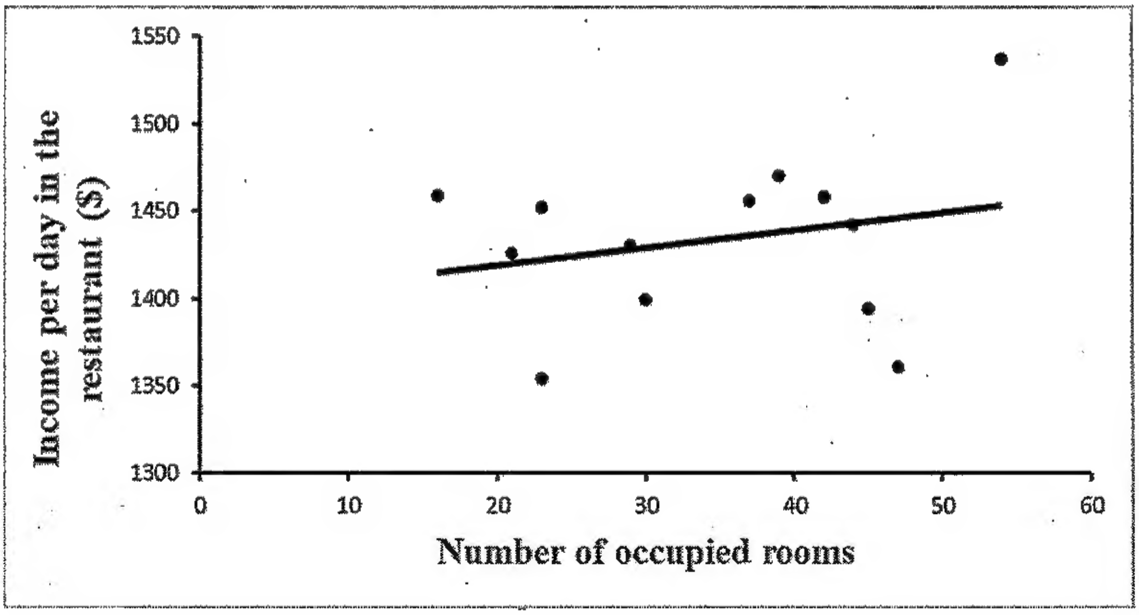
(10 marks)

Continued…

Question 3 (30 Marks)

(a) A suburban hotel derives its gross income from its hotel and restaurant operations. The owners are interested in the relationship between the number of rooms occupied on a nightly basis and the revenue per day in the restaurant. The collected data, scatterplot and summary output of the relationship between the two variables are shown below:

Number of occupied rooms	Income per day in the restaurant ($)
23	1452
47	1361
21	1426
39	1470
37	1456
29	1430
23	1354
44	1442
45	1394
16	1459
30	1399
42	1458
54	1537



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SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8449
R Square	0.7139
Adjusted R Square	0.7524
Standard Error	49.4464
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1716.332	1716.332	0.701991	0.419948
Residual	11	26894.44	2444.949		
Total	12	28610.77			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	1398.49	44.20264	31.63804	3.74e-12
Occupied Rooms	1.0171	1.213952	0.837849	0.419948

(i) State the dependent variable and the independent variable for the above regression model.

(2 marks)

(ii) Write the least square regression line for the above relationship between the two variables. State the unit of measurement for each variable.

(4 marks)

(iii) What do the coefficient of the regression line tells you about the relationship between the number of occupied rooms and the income of the restaurant?

(3 marks)

(iv) Determine the coefficient of correlation and discuss the role of this coefficient value for this model. **(4 marks)**

(v) Predict the income of the restaurant if the number of occupied rooms was 55. Is this estimation reliable? Explain.

(4 marks)

(vi) State the coefficient of determination and describe what it tells you.

(3 marks)

Continued...

b) Bill Simpson, owner of a California vineyard has collected the following information describing the prices and quantities of harvested crop for year 2003 and 2013:

Type of grape	2003		2013	
	Price ($)	Quantity	Price ($)	Quantity
Ruby Cabernet	108	1280	111	1360
Barbera	93	830	101	890
Chenin Blanc	97	1640	107	1460
Pinot Noir	82	520	79	480

Compute and interpret the Laspeyres Price Index (LPI) and Paashe Price Index (PPI) for 2013 using 2003 as the base period.

(10 marks)

End of Questions.

A. STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\bar{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B)   if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution B(n, p) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) =
$\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{p^\wedge}$

Where $\sigma_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\dfrac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \dfrac{(Z_{\alpha/2})^2\, \hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$
One Sample Proportion Test	
$z = \dfrac{\hat{p}-p}{\sigma_p}$ where $\sigma_p = \sqrt{\dfrac{p(1-p)}{n}}$	
Two Sample Mean Test	
Standard Deviation (σ) Known $z = \dfrac{(\overline{x}_1-\overline{x}_2)-(\mu_1-\mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	**Standard Deviation (σ) Not Known** $t = \dfrac{(\overline{x}_1-\overline{x}_2)-(\mu_1-\mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{(n_1+n_2-2)}$
Two Sample Proportion Test	
$Z = \dfrac{\hat{p}_1-\hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $p = \dfrac{X_1+X_2}{n_1+n_2}$ *where X_1 and X_2 are the number of successes from each population*	

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	1	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

Where p= number of predictor

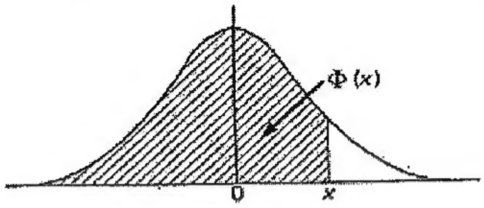
G. INDEX NUMBERS

Simple Price Index	**Laspeyres Quantity Index**
$P = \frac{p_t}{p_0} \times 100$	$P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index	**Paasche Quantity Index**
$P = \frac{\sum p_t}{\sum p_0}(100)$	$P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index	**Fisher's Ideal Price Index**
$P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	$\sqrt{(\text{Laspeyres Price Index})(\text{Paasche Price Index})}$
Paasche Price Index	**Value Index**
$P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	$V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

B. STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

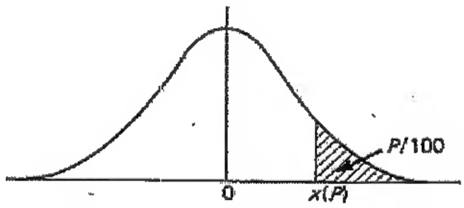
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteq \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



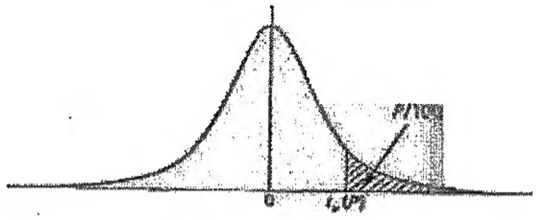
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1 + t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291